AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TYJ MATHEMATICS SOLUTION 10 NOV 2019

41. (d) Here the centre of circle $(3, -1)$ must lie on the line $x + 2by + 7 = 0$.

Therefore, $3 - 2b + 7 = 0 \Rightarrow b = 5$.

42. (a) Radius of circle is $\left|\frac{2+3-4}{\sqrt{5}}\right| = \frac{1}{\sqrt{5}}$

Therefore, equation is $(x-1)^2 + (y+3)^2 = \frac{1}{5}$

or $x^2 + y^2 - 2x + 6y + 1 + 9 = \frac{1}{5}$

or $5x^2 + 5y^2 - 10x + 30y + 49 = 0$.

43. (b) Centre of circle = Point of intersection of diameters = (1, –1)

Now area $= 154 \Rightarrow \pi r^2 = 154 \Rightarrow r = 7$

Hence the equation of required circle is

$(x-1)^2 + (y+1)^2 = 7^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47$.

44. (b) Centre (1, 2) and since circle touches x-axis, therefore, radius is equal to 2.

Hence the equation is $(x-1)^2 + (y-2)^2 = 2^2$

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$.

Trick : The only circle is $x^2 + y^2 - 2x - 4y + 1 = 0$, whose centre is (1, 2).

45. (c) $2\sqrt{g^2 - c} = 2a$ ....(i)

$2\sqrt{f^2 - c} = 2b$ ....(ii)

On squaring (i) and (ii) and then subtracting (ii) from (i), we get $g^2 - f^2 = a^2 - b^2$.

Hence the locus is $x^2 - y^2 = a^2 - b^2$.

46. (b) The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and $3x - 4y - \frac{7}{2} = 0$ and so it is equal to $\frac{4}{\sqrt{9+16}} + \frac{7/2}{\sqrt{9+16}} = \frac{3}{2}$. Hence radius is $\frac{3}{4}$.

47. (a) Circle is $x^2 + y^2 - 2x - 2y + 1 = 0$ as centre is (1, 1) and radius = 1.

48. (c) Centre is (2, 3). One end is (3, 4).

P_2 divides the join of P_1 and O in ratio of 2 : 1.

Hence P_2 is $\left(\frac{4-3}{2-1}, \frac{6-4}{2-1}\right) \equiv (1, 2)$.

49. (c) In the equation of circle, there is no term containing xy and coefficient of x^2 and y^2 are equal. Therefore $2 - q = 0 \Rightarrow q = 2$ and $p = 3$.

50. (c) Here $2\sqrt{g^2 - c} = 2a \Rightarrow g^2 - a^2 - c = 0$ .....(i)

and it passes through (0, b), therefore

$b^2 + 2fb + c = 0$ ....(ii)

On adding (i) and (ii), we get $g^2 + 2fb = a^2 - b^2$

Hence locus is $x^2 + 2by = a^2 - b^2$.

51. (b) First find the centre. Let centre be (h, k), then

$\sqrt{(h-2)^2 + (k-3)^2} = \sqrt{(h-4)^2 + (k-5)^2}$ ....(i)

and $k - 4h + 3 = 0$ ....(ii)

From (i), we get $-4h - 6k + 8h + 10k = 16 + 25 - 4 - 9$

or $4h+4k-28=0$ or $h+k-7=0$ ....(iii)

From (iii) and (ii), we get (h, k) as (2, 5). Hence centre is (2, 5) and radius is 2. Now find the equation of circle.

Trick : Obviously, circle $x^2+y^2-4x-10y+25=0$ passes through (2, 3) and (4, 5).

52. (b) Centre is $(-4,-5)$ and passes through (2, 3).

53. (c) Equation of circle passing through (0, 0) is

$x^2+y^2+2gx+2fy=0$ ....(i)

Also, circle (i) is passing through (0, b) and (a, b)

$\therefore f=-\frac{b}{2}$ and $a^2+b^2+2ag+2\left(-\frac{b}{2}\right)b=0$

$\Rightarrow g=-\frac{a}{2}$

Hence the equations of circle is, $x^2+y^2-ax-by=0$.

54. (a) Equation of circle concentric to given circle is $x^2+y^2-6x+12y+k=0$ ....(i)

Radius of circle (i) $=\sqrt{2}$ (radius of given circle)

$\Rightarrow \sqrt{9+36-k}=\sqrt{2}\sqrt{9+36-15}$

$\Rightarrow 45-k=60 \Rightarrow k=-15$

Hence the required equation of circle is

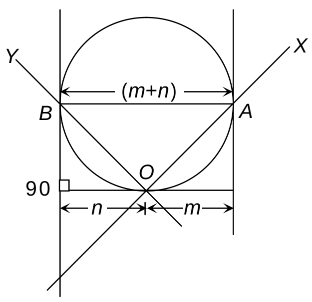
$x^2+y^2-6x+12y-15=0$.

55. (d) See condition for circle and also condition for circle to pass through origin *i.e.* origin satisfies equation of circle or $c=0$.

56. (a) Centre (3, –1). Line through it and origin is $x+3y=0$.

57. (c) As centres lie on angle bisectors of co-ordinate axes or $x=0$ and $y=0$, we get two lines which are perpendicular to each other on which the centres lie *i.e.* $x=y$ and $x=-y$ or $x^2-y^2=0$ as combined equation.

58. (b) It is clear from the figure that diameter is $m+n$.



59. (a) Let its centre be (h, k), then $h-k=1$ ...(i)

Also radius $a=3$

Equation is $(x-h)^2+(y-k)^2=9$

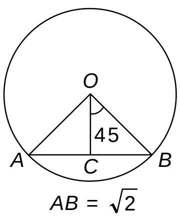
Also it passes through (7, 3)

i.e., $(7-h)^2+(3-k)^2=9$ ....(ii)

We get h and k from (i) and (ii) solving simultaneously as (4, 3). Equation is $x^2+y^2-8x-6y+16=0$.

Trick : Since the circle $x^2+y^2-8x-6y+16=0$ satisfies the given conditions.

60. (c) Let AB be the chord of length $\sqrt{2}$, O be centre of the circle and let OC be the perpendicular from O on AB. Then



$AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

In $\Delta\, OBC, OB = BC \operatorname{cosec} 45^o = \frac{1}{\sqrt{2}}.\sqrt{2} = 1$

$\therefore$ Area of the circle $= \pi(OB)^2 = \pi$.